Code No: R059210101 **Set No. 1**

II B.Tech I Semester Regular Examinations, November 2007
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Find the value of K such that the rank of $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & K \end{bmatrix}$ is 2

   (b) Determine whether the following equations will have a non-trivial solution if so solve them.
   3x + 4y - z - 6ω = 0; 2x + 3y + 2z - 3ω = 0
   2x + y - 14z - 9ω = 0; x + 3y + 13z + 3ω = 0. [8+8]

2. (a) Define eigen value and eigen vector of a matrix A. Show that trace of A equals to the sum of the eigen values of A.

   (b) Verify that the sum of eigen values is equal to the trace of A for the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find the corresponding eigen vectors. [8+8]

3. (a) Prove that the eigen values of a skew Hemitian matrix are either zero or purely imaginary.

   (b) Find the nature of the quadratic form $2x^2 + 2y^2 + 2z^2$ + 2yz. Also find Rank, index and signature. [8+8]

4. (a) Expand f(x) = cos ax as a Fourier series in (-π, π) where a is not an integer. Hence prove that $\cot\theta = \frac{1}{\theta} + \frac{2\theta}{\theta^2-\pi^2} + \frac{2\theta}{\theta^2-4\pi^2} + ...$

   (b) If $f(x) = x, 0 < x < \frac{\pi}{2}$
   $= \pi - x, \ \frac{\pi}{2} < x < \pi$
   Show that $f(x) = \frac{4}{\pi}\left[\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \ ....\right]$. [8+8]

5. (a) Form the partial differential equation by eliminating the arbitrary function from z = y f (x^2 + z^2).

   (b) Solve the partial differential equation z(x-y) = px^2 - qy^2

   (c) Solve the partial differential equation (x-y)p + (y-x-z)q = z. [5+5+6]

6. The temperature at one end of a bar is 50 cm long with insulated sides is kept at 0^o c and that the other end is kept at 100^o c until steady state condition prevails. The two ends are then suddenly insulated so that the temperature gradient is zero at each end thereafter. Find the temperature distribution. [16]

7. (a) Find the Fourier sine transform of $\frac{1}{x(a^2+x^2)}$

   (b) Find the finite sine and cosine transform of
   $f(x) = 1 \, in \, 0 < x < \pi/2$
   $\quad = -1 \, in \, \pi/2 < x < \pi.$ [10+6]

8. (a) Find $Z[(n+1)^2]$

   (b) Solve the difference equation using z-transforms u_{n+2} - $5u_{n+1} + 6u_n = 4^n$ given that u_0 =0 $u_1 = 1$. [6+10]

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Code No: R059210101 **Set No. 2**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the rank of the matrix by reducing it to the echlon form
$$\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$$
   (b) Show that the equations
   3x + 4y + 5z = a, 4x + 5y + 6z = b
   5x + 6y + 7z = c, do not have a solution unless a + c = 2b. [8+8]

2. (a) Find the characteristic roots of the matrix and the corresponding eigen values
$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
   (b) If λ_1, λ_2,........, λ_n are the eigen values of A, then prove that the eigen values of
   (A - kI) are $\lambda_1 - k,\ \lambda_2 - k,\ \lambda_3 - k, ...,\ \lambda_n - k.$ [10+6]

3. Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is a skew-Hermitian matrix and also umitary Find eigen values and the corresponding eigen vectors of A. [16]

4. (a) Find a Fourier series to represent x − x^2 from x = $-\pi$ to x =π.. Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$
   (b) Find the half range sine series for the function
   $$\text{f (x)} = \begin{cases} \frac{1}{4} - x, 0 < \text{x} < \frac{1}{2} \\ x - \frac{3}{4}, \frac{1}{2} < \text{x} < 1 \end{cases}$$ [10+6]

5. (a) Form the partial differential equations by eliminating the arbitrary functions $\text{f}(\text{x} + \text{y} + \text{z}, \text{x}^2 + y^2 + z^2) = 0$
   (b) Solve the partial differential equation $2z^4P^2 - x + z^2\, q + y = 0$
   (c) Solve the partial differential equation $p^2\, q^2 + x^2\, y^2 = x^2\, q^2\, (x^2 + y^2)$.[5+6+5]

6. Solve ∂^2u/∂x^2+∂^2u/∂y^2= 0. Subject to the boundary conditions u(0,y) = u(L,y) = u(x,L)=0 and u(x,0) = sin n Πx/L. [16]

7. (a) State and prove Fourier Integral Theorem.

   (b) Find the Fourier transform of $f(x) = \begin{cases} e^{ikx} & a < x < b \\ 0 & x < a \; and \; x > b \end{cases}$ [8+8]

8. (a) If $Z\left[u_n\right] = \frac{z^2+2z+6}{(z-1)^3}, \; |z| \geq$ then find $u_o, \, u_1 \, and \, u_2$

   (b) Solve using z transforms the difference equation $u_{n+2} + 2u_{n+1} + u_n =$ n given that $u_o = u_1 = 0$. [8+8]

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Code No: R059210101 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the value of K such that the rank of $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & K \end{bmatrix}$ is 2

   (b) Determine whether the following equations will have a non-trivial solution if so solve them.
   3x + 4y - z - 6ω = 0; 2x + 3y + 2z - 3ω = 0
   2x + y - 14z - 9ω = 0; x + 3y + 13z + 3ω = 0. [8+8]

2. (a) Find the eigen values and the corresponding eigen vectors of the matrix
   $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

   (b) Prove that the product of eigen values of a matrix is equation its determinant. [10+6]

3. (a) Prove that the eigen values of a real symmetric matrix are real.

   (b) Reduce the quadtatic form $7x^2 + 6y^2 + 5z^2$ - 4xy - 4yz to the canonical form. [6+10]

4. (a) Given that $f(x) = x + x^2$ for $-\pi <$ x $< \pi$ find the Fourier expansion of f(x). Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$

   (b) Find the half range sine series for f (x) = x (π − x) , in0 < x < π. Deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + ... = \frac{\pi^3}{32}$. [10+6]

5. (a) Form the partial differential equation by eliminating the arbitrary function from z = y f (x^2 + z^2).

   (b) Solve the partial differential equation z(x-y) = px^2 - qy^2

   (c) Solve the partial differential equation (x-y)p + (y-x-z)q = z. [5+5+6]

6. Solve the boundary value problem $u_t = u_{xx}$; 0 < x <ℓ, t > 0 with u (0, t) = 0 ; u_x (ℓ, t) = 0 and u (x, 0) =x. [16]

7. (a) Find the finite Fourier sine and cosine transforms of
      i. f (x) = x in (0, l).

   (b) Find the finite sine and transform of f(x) = cos kx in $0 < x < \pi$ [8+8]

8. (a) If $z\,[n] = \frac{z}{(z-1)^2}$, find z[n + 2]

   (b) Solve the difference equation, using Z - transforms
   y_{n+2} - $4y_{n+1}$ + $3y_n = 0$ given that $y_0 = 2$ and $y_1 = 4$. [8+8]

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Code No: R059210101

Set No. 4

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Determine the rank of the matrix.
$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$ by reducing it to normal form.
(b) Find whether the following equations are consistent, if so solve them.
x + 2y - z = 3
3x - y + 2z = 1
2x - 2y + 3z = 2
x - y + z = -1. [8+8]

2. Define a modal matrix, Diagonalize $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ [16]

3. (a) If $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$ show that A is Hermitian and iA is skew-Hermitian matrices.
(b) Identify the nature of the quadratic form
$-3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$. Find index and signature. [8+8]

4. (a) Find the Fourier series to represent f(x) = x^2 - 2, when $-2 \leq$ x ≤ 2
(b) Obtain a half range cosine series for $f(x) = \begin{cases} kx, 0 \leq x \leq \frac{L}{2} \\ k(L-x), \frac{L}{2} \leq x \leq L \end{cases}$
Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$........ [10+6]

5. (a) Form the partial differential equation by eliminating the arbitrary function from z = y f (x^2 + z^2).
(b) Solve the partial differential equation z(x-y) = px^2 - qy^2
(c) Solve the partial differential equation (x-y)p + (y-x-z)q = z. [5+5+6]

6. (a) $4u_x + u_y$ = 3u given u = $3e^{-y}$ - e^{-5y} when x = 0.
(b) Find the general solution of one-dimensional heat equation. [8+8]

7. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$

   Hence evaluate $\int\limits_0^\infty \left[\frac{x\cos x - \sin x}{x^2}\right] \cos\frac{x}{2}\ dx.$

   (b) Find Fourier cosine transform of $f(x) = \begin{cases} \cos x & 0 < x < a \\ 0 & x \geq a \end{cases}$ [10+6]

8. (a) If $Z\,[u_n] = \frac{z^2+2z+6}{(z-1)^3},\ |z| \geq$ then find $u_o,\ u_1\,and\,u_2$

   (b) Solve using z transforms the difference equation $u_{n+2} + 2u_{n+1} + u_n =$ n given that $u_o = u_1 = 0.$ [8+8]

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